CHANDIGARH UNIVERSITY

Discover. Learn. Empower.

INSTITUTE - UIE

Bachelor of Engineering (Computer Science & Engineering)

Subject Name : CALCULUS & VECTOR SPACES

Subject Code : 20SMT-175

BE :CSE(All IT branches)

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Introduction to Inner Product Spaces

Course Outcomes

CO Number	Title	Level
CO1	1) The concept of partial derivatives and its application in real life situations 2) The concept of Multiple Integrals and its applications.	**Remember & Understand**
CO2	The concept of Group theory and its application of analysis to Engineering problems.	**Remember & Understand**
CO3	The concept of vector spaces in a comprehensive manner.	**Remember & Understand**

- Course prerequisites

➢ Basic Knowledge of Sets.

➢ Basic Knowledge of **binary operations**.

➢ Basic Knowledge of functions.

➢ Basic Knowledge of vectors.

Topic Outcomes

- Students will able to understand basic concept of inner product spaces
- Students will able to understand basic concept orthogonal and orthogonal sets.
- Students now able to apply Gram-Schmidt Orthogonalization Process for orthonormal basis.

Inner-Product Spaces

A vector space V over F is called an inner product space if there is a function $f: V \times V \longrightarrow F$ satisfying the following conditions:

For $u, v, w \in V; \alpha, \beta \in F$

(i) $f(u,v) = \overline{f(v,u)}$

(ii) $f(u,u) \geq 0 \ and \ f(u,u) = 0 \Leftrightarrow u = 0$

(iii) $f(\alpha u + \beta v, w) = \alpha f(u,w) + \beta f(v,w)$

$f(u,v)$ is generally denoted by any one of the following symbols

$$(u,v); u.v; < u|v >$$

Inner-Product Spaces

Results:

(1) In an inner product space V ; for all $u,v,w,x \in V; \alpha,\beta,\gamma,\delta \in F$

(i) $<u|\alpha v+\beta w> = \overline{\alpha}<u|v> + \overline{\beta}<u|w>$

(ii) $<\alpha u+\beta v|\gamma w+\delta x> = \alpha\overline{\gamma}<u|v> + \alpha\overline{\delta}<u|x> + \beta\overline{\gamma}<v|w> + \beta\overline{\delta}<v|x>$

(iii) $<0|v> = <u|0> = 0$

(iv) $<u|v> = 0 \forall v \in V \Rightarrow u = 0$ and $<u|v> = 0 \forall u \in V \Rightarrow v = 0$

(v) $\|\alpha u\| = |\alpha|\|u\|$

(2) For all $u,v \in V, |<u|v>| \leq \|u\|.\|v\|$

(3) Every inner product space is a metric space.

Inner-Product Spaces

Examples:

- The space of n-dimensional arrays with real coefficients is an inner product space.
  If $v=(v_1,\ldots,v_n)$ and $w=(w_1,\ldots,w_n)$ then:
  $$\langle v,w\rangle=v_1w_1+\ldots+v_nw_n$$
- If M is a symmetric matrix ($M=M^t$) whose eigen-values are all positive, then the space of n-dimensional arrays with real coefficients is an inner product space.
  If $v=(v_1,\ldots,v_n)$ and $w=(w_1,\ldots,w_n)$ then:
  $$\langle v,w\rangle_M=vMw^t$$

Inner-Product Spaces

Examples:

- The space of m x n matrices with real coefficients is an inner product space.
  If M and N are two m x n matrices then:

$$\langle M , N\rangle = \text{Trace}(M^t N)$$

- The spaces of real-valued functions defined in 1D, 2D, 3D,… are real inner product space.
  If f and g are two functions in 1D, then:

$$\langle f, g\rangle = \int_{-\infty}^{\infty} f(x)g(x)dx$$

- The spaces of real-valued functions defined on the circle, disk, sphere, ball,… are real inner product spaces.
  If f and g are two functions defined on the

$$\langle f, g\rangle = \int_{0}^{2\pi} f(\theta)g(\theta)d\theta$$

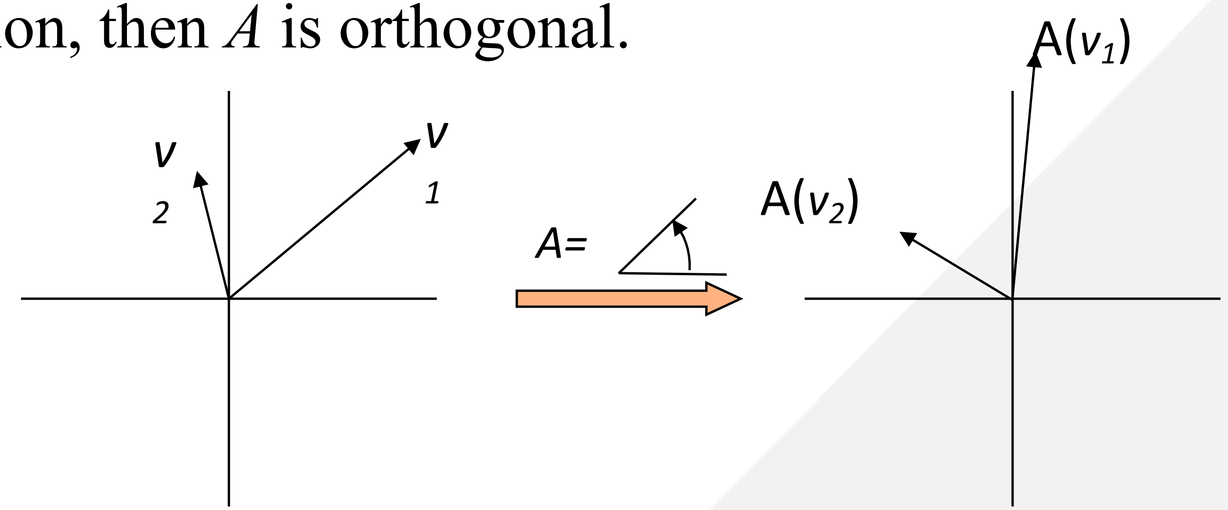
Inner-Product Spaces

Orthogonal / Unitary Operators

If V is a real / complex inner product space, then a linear map $A: V \to V$ is orthogonal / unitary if it preserves the inner product as $\langle v , w\rangle = \langle Av , Aw\rangle$

for all $v, w \in V$.

Examples: If V is the space of real, two-dimensional, vectors and A is any rotation or reflection, then A is orthogonal.



Inner-Product Spaces

Orthogonal vector:

$u \in V$ is said to be orthogonal to $v \in V$ if $< u|v \geq 0$.

Orthogonal complement:

If W is a subspace of V, then an orthogonal complement of W (denoted by $W^{\perp}$ defined by $\{u \in V: < u|v \geq 0 \ \forall\ w \in W\}$

Note: $W^{\perp^{\perp}} =$ W.

Orthogonal set:

A subset X of V is said to be an orthogonal set if for any $x, y \in X$, $< x|y >=$ 0 whenever $x \neq y$.

Inner-Product Spaces

Orthonormal set:

A subset X of V is called an orthonormal set if

(i) $||x|| = 1 \;\; \forall \;\; x \in X$

(ii) For any $x, y \in X$, $< x|y >= 0$ whenever $x \neq y$.

Results:

- Any orthogonal set of nonzero vectors is linearly independent.
- **(Gram-Schmidt Orthogonalization Process)** Every finite dimensional inner product space has an orthonormal basis.
- For each subspace W of a finite-dimensional inner product space V,

$$V = W \oplus W^{\perp}$$

References

Reference Books

- Elements of Discrete Mathematics, (Second Edition) C. L. Liu, McGraw Hill, New Delhi, 2017
- Graph Theory with Applications, J. A. Bondy and U. S. R. Murty, Macmillan Press, London.
- Topics in Algebra, I. N. Herstein, John Wiley and Sons. Digital Logic & Computer Design, M. Morris Mano, Pearson.

Online Video Sites:

1. NPTEL
2. Coursera
3. Unacademy

THANK YOU